Journal of Engineering Physics and Thermophysics, Vol. 80, No. 1, 2007

HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

SELF-SIMILAR SOLUTION OF THE PROBLEM OF A STRONG EXPLOSION IN A PERFECT GAS. LAGRANGIAN DESCRIPTION

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The problem of a strong one-dimensional explosion in a perfect gas has been considered. The analytical solution of self-similar equations describing the dynamics of the explosion and the propagation of a strong shock wave has been obtained in Lagrangian mass coordinates.

Introduction. Investigation of the propagation of strong shock waves generated by an explosion is a classical gas-dynamics problem at present. Strong-explosion theory derives from the necessity of describing the processes of motion of a continuum and a shock wave in the explosion of a charge of high specific energy and small weight and volume. This theory was developed in the works of L. I. Sedov, J. von Neumann, G. Taylor, and K. P. Stanyukovich [1–7]. Different aspects of strong-explosion theory were the focus of numerous works reviewed in [8]. The analytical solution of the corresponding self-similar equations in Eulerian coordinates was given in [1–2].

At the same time, it is common knowledge that one-dimensional nonstationary flows of the explosion type, particularly, numerical solution of the corresponding gas-dynamics equations, are conveniently investigated in Lagrangian variables. This approach enables one to naturally describe contact discontinuities. In this case it is much more simple to consider the chemical reactions and the kinetics of ionization and recombination processes in high-temperature explosion products and in the ambient medium [9, 10]. Radiative transfer can also be successfully analyzed in such an approach [11]. The present work gives the solution of the problem of a strong explosion in a perfect gas in Lagrangian mass coordinates.

Formulation of the Problem. We consider a one-dimensional strong explosion in a medium with a constant density ρ_0. The gas motion is known to be described by the following equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial r} + (\nu - 1)\frac{\rho u}{r} = 0 , \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 , \tag{1}$$

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon + \frac{u^2}{2}\right)\right] + \frac{1}{r^{\nu-1}}\frac{\partial}{\partial r}\left(r^{\nu-1}\rho u\left(\varepsilon + \frac{p}{\rho} + \frac{u^2}{2}\right)\right) = 0 . \tag{2}$$

Equations (1) express the laws of conservation of mass and momentum, whereas Eq. (2) expresses the law of conservation of energy. In them, ν is the symmetry factor (1, 2, and 3 respectively for plane, cylindrical, and spherical cases). If we disregard mechanisms of energy transfer other than a hydrodynamic mechanism, the relation

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + u\frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) = 0 , \tag{3}$$

expressing the constancy of the entropy of a gas particle holds true behind the shock-wave front. The specific internal energy ε is determined by the gas equation with a constant adiabatic exponent γ

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1062-0125/07/8001-0089

$$\varepsilon = \frac{p/\rho}{\gamma - 1} . \tag{4}$$

In the case of a strong explosion the gas pressure in an unperturbed region can be disregarded compared to the pressure behind the shock-wave front ($p_0 = 0$). If the energy release is considered to be instantaneous and the mass of the explosion charge is assumed to be zero, the motion will be dependent on just two parameters: energy E (explosion energy per unit area in a plane case and per unit length for cylindrical symmetry and the total energy for spherical geometry) and ρ_0. Their dimensions are J/m$^{(3-\nu)}$ (for E) and kg/m^3 (for ρ_0). Therefore, the only dimensionless combination that can be composed of the quantities r and t and the parameters E and ρ_0 is

$$\eta = \frac{r}{r_f} , \quad r_f = \alpha \, (E/\rho_0)^{1/(\nu+2)} \, t^{2/(\nu+2)} . \tag{5}$$

The proportionality factor α is found from the law of conservation of energy. The external boundary of the region involved in motion is the shock-wave front on which the relations for a strong discontinuity hold [1]:

$$r = r_f : \quad \rho_f = \frac{\gamma + 1}{\gamma - 1} \rho_0 , \quad u_f = \frac{2}{\gamma + 1} D , \quad p_f = \frac{2}{\gamma + 1} \rho_0 D^2 . \tag{6}$$

The wave velocity D is related to the coordinate of the front by the relation

$$D = \frac{dr_f}{dt} = \frac{2}{\nu + 2} \frac{r_f}{t} . \tag{7}$$

In the plane (on the axis or at the center) of symmetry, the mass velocity, apparently, vanishes:

$$r = 0 : \quad u = 0 . \tag{8}$$

Lagrangian Equations of Strong Explosion. Below, we consider the solution of this problem in Lagrangian variables. We introduce the mass coordinate according to the relation

$$dm = \sigma_\nu \, \rho r^{\nu-1} dr , \tag{9}$$

where σ_ν = 2, 2π, and 4π for ν = 1, 2, and 3 respectively. If m is reckoned off from the plane (axis or center) of symmetry, the mass coordinate of a point with an Eulerian coordinate r is the mass of the gas in the domain [0, r]. In mass coordinates, gas-dynamics equations have the form

$$\frac{1}{\rho} = \sigma_\nu r^{\nu-1} \frac{\partial r}{\partial m} , \quad \frac{\partial u}{\partial t} + \sigma_\nu r^{\nu-1} \frac{\partial p}{\partial m} = 0 , \quad u = \frac{\partial r}{\partial t} , \tag{10}$$

$$\frac{\partial}{\partial t} \left(\varepsilon + \frac{u^2}{2} \right) + \sigma_\nu \frac{\partial}{\partial m} \left(r^{\nu-1} p u \right) = 0 , \quad \frac{\partial}{\partial t} \left(\frac{p}{\rho^\gamma} \right) = 0 . \tag{11}$$

In (10) and (11), by the symbol $\partial/\partial t$ we mean the substantial time derivative $\partial/\partial t + u\partial/\partial r$. We introduce the mass coordinate of the shock-wave front. Since the wave moves in the medium with a constant density, in accordance with (5) and (9) we have

$$m_f = \frac{\sigma_\nu}{\nu} \rho_0 r_f^\nu = \beta \frac{\sigma_\nu}{\nu} \left(\rho_0^2 E^\nu \right)^{1/(\nu+2)} t^{2\nu/(\nu+2)} . \tag{12}$$

Here the coefficient β is related to the factor α from (5) by the relation $\beta = \alpha^\nu$.

Since the pressure (and energy) of the gas ahead of the shock-wave front can be disregarded in the strong-explosion approximation, the law of conservation of total energy in the region of motion yields

$$E = \int_0^{m_f} \left(\varepsilon + \frac{u^2}{2} \right) dm . \tag{13}$$

The self-similar dimensionless mass coordinate is determined as

$$\xi = \frac{m}{m_f} . \tag{14}$$

Functions describing gasdynamic flow can be represented in the form

$$r = r_f \, \eta (\xi) , \quad u = \frac{2}{\gamma + 1} DU (\xi) , \quad \rho = \frac{\gamma + 1}{\gamma - 1} \rho_0 G (\xi) , \quad p = \frac{2}{\gamma + 1} \rho_0 D^2 P (\xi) . \tag{15}$$

Introducing the specific volume $v = \rho^{-1} = \rho_0^{-1}(\gamma - 1)/(\gamma + 1)V(\xi)$ which is the reciprocal of density and substituting (15) into (10) and (11), we obtain a system of differential equations for the dimensionless representative functions V, U, P, and η:

$$\frac{\gamma - 1}{\gamma + 1} V = \nu \eta^{\nu - 1} \frac{d\eta}{d\xi} , \quad U + 2\xi \frac{dU}{d\xi} = 2\eta^{\nu - 1} \frac{dP}{d\xi} , \quad \frac{2}{\gamma + 1} U = \eta - \nu \xi \frac{d\eta}{d\xi} , \tag{16}$$

$$\frac{d}{d\xi} \left[\xi \, (PV + U^2) \right] = 2 \frac{d}{d\xi} (\eta^{\nu - 1} PU) , \quad \frac{d}{d\xi} (\xi P V^{\gamma}) = 0 . \tag{17}$$

Boundary conditions for the self-similar functions, according to (6), (8), and (15), have the form

$$\xi = 0 : \ U = 0 ; \ \xi = 1 : \ \eta = 1 , \ U = 1 , \ P = 1 , \ V = 1 . \tag{18}$$

Analysis and Solution. Relations (17) and (18) yield two algebraic equations expressing the laws of conservation of energy and the constancy of the entropy of a gas particle behind the shock-wave front:

$$\xi \, (PV + U^2) = 2\eta^{\nu - 1} PU , \quad \xi P V^{\gamma} = 1 . \tag{19}$$

Thus, the problem is reduced to determination of the solution of Eqs. (16) and (19), satisfying conditions (18). To analyze the given system we introduce the dimensionless temperature Z:

$$Z = PV . \tag{20}$$

Using it and relations (19) we eliminate the variables V, P, and η from (16):

$$V = (\xi Z)^{-1/(\gamma - 1)} , \quad P = Z \, (\xi Z)^{1/(\gamma - 1)} , \quad \eta^{\nu - 1} = \frac{Z + U^2}{2Z^2 U} (\xi Z)^{-(2 - \gamma)/(\gamma - 1)} . \tag{21}$$

We note that in the case of plane symmetry ($\nu = 1$) where the last equation of (21) does not contain an Eulerian coordinate, the quantity η, according to (16), is related to U and V by the dependence

$$\eta = \frac{2}{\gamma + 1} \left(U + \frac{\gamma - 1}{2} \xi V \right) . \tag{22}$$

Finally Eqs. (16) are reduced to a system of two differential equations

$$\frac{dU}{d\ln\xi} = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}\,, \quad \frac{dZ}{d\ln\xi} = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}\,, \tag{23}$$

the coefficients on the right-hand side of which are expressed as follows:

$$a_1 = \nu\left[(Z+U^2)^2 + (\gamma-1)\,Z\,(Z-U^2)\right],\quad b_1 = \nu\,(\gamma-1)\,U^3\,,\quad c_1 = (\gamma-1)\,ZU\,(Z+U^2)\,,$$
$$a_2 = 2\,(\gamma-1)\,ZU\,,\quad b_2 = -\,\gamma\,(Z+U^2)\,,\quad c_2 = Z^2 + (2-\gamma)\,ZU^2\,. \tag{24}$$

Further analysis is standard. Dividing the second equation from (23) by the first equation, we obtain

$$\frac{dZ}{dU} = \frac{\nu\left[(Z+U^2)^2 + (\gamma-1)\,Z\,(Z-U^2)\right]\left[Z^2 + (2-\gamma)\,ZU^2\right] - 2\,(\gamma-1)^2\,Z^2U^2\,(Z+U^2)}{-\,\gamma\,(\gamma-1)\,ZU\,(Z+U^2)^2 - \nu\,(\gamma-1)\,U^3\left[Z^2 + (2-\gamma)\,ZU^2\right]}\,. \tag{25}$$

To simplify this equation we introduce a new variable y:

$$Z = U^2 y\,. \tag{26}$$

Substituting (26) into (25) and carrying out transformations, we write the following equation:

$$\frac{1}{U}\frac{dU}{dy} = \frac{(\gamma-1)\left[\gamma\,(y+1)^2 + \nu\,(y+2-\gamma)\right]}{d_0 y^3 + d_1 y^2 + d_2 y + d_3}\,, \tag{27}$$

where

$$d_0 = -\,\gamma\,[2\,(\gamma-1)+\nu]\,;\quad d_1 = -\left[4\gamma\,(\gamma-1) + 2\nu\,(\gamma-1) + \nu\left\{\gamma\,(2-\gamma) + 3 - \gamma\right\} - 2\,(\gamma-1)^2\right];$$
$$d_2 = -\left[2\gamma\,(\gamma-1) + 2\nu\,(\gamma-1)\,(2-\gamma) + \nu\left\{1 + (3-\gamma)\,(2-\gamma)\right\} - 2\,(\gamma-1)^2\right];\quad d_3 = -\,\nu\,(2-\gamma)\,. \tag{28}$$

Despite its complex form, Eq. (27) is solved quite easily, since the polynomial in the denominator is factored. As a result we have

$$\frac{1}{U}\frac{dU}{dy} = -\,(\gamma-1)\,\frac{\gamma y^2 + (2\gamma+\nu)\,y + \nu\,(2-\gamma) + \gamma}{(y+1)\,(\gamma y+1)\left\{[2\,(\gamma-1)+\nu]\,y + \nu\,(2-\gamma)\right\}}\,. \tag{29}$$

The solution of the last equation, which satisfies the condition on the shock-wave front ($U = 1$ for $y = 1$), has the form

$$U = \left(\frac{y+1}{2}\right)^{\frac{\nu}{\nu+2}} \left(\frac{\gamma+1}{\gamma y+1}\right)^{\frac{(\gamma-1)(\nu-1)}{(\gamma-1)\nu+2}} \left(\frac{2\,(\gamma-1) + \nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y + \nu\,(2-\gamma)}\right)^{\frac{\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)}{(\nu+2)[2(\gamma-1)+\nu][(\gamma-1)\nu+2]}}\,. \tag{30}$$

Thereafter, from (26), we immediately determine the dependence $Z(y)$:

$$Z = y\left(\frac{y+1}{2}\right)^{\frac{2\nu}{\nu+2}} \left(\frac{\gamma+1}{\gamma y+1}\right)^{\frac{2(\gamma-1)(\nu-1)}{(\gamma-1)\nu+2}} \left(\frac{2\,(\gamma-1) + \nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y + \nu\,(2-\gamma)}\right)^{\frac{2[\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)]}{[2(\gamma-1)+\nu][(\gamma-1)\nu+2](\nu+2)}}\,. \tag{31}$$

Expressions (30) and (31) represent the parametric solutions of the Lagrangian strong-explosion equations for mass velocity and temperature. Now we direct our attention to solution of the first equation of system (23). Since we have $\frac{dU}{d\ln\xi} = \frac{dU}{dy}\frac{dy}{d\ln\xi}$ and can use formula (29) for $\frac{dU}{dy}$, we find the differential equation relating y and ξ:

$$\frac{1}{\xi}\frac{d\xi}{dy} = -\nu\,\frac{\gamma y^2 + 2y + \gamma}{y\,(y+1)\left\{[2\,(\gamma-1)+\nu]\,y + \nu\,(2-\gamma)\right\}}\,. \tag{32}$$

The solution of this equation is fundamentally different for the adiabatic exponents $\gamma \neq 2$ and $\gamma = 2$, which is due to the appearance of a multiple root in the polynomial in the denominator of (32) in the latter case. The solution has the form

$$\xi = \begin{cases} y^{\frac{\gamma}{\gamma-2}}\left(\frac{2}{y+1}\right)^{\frac{2\nu}{\nu+2}}\left(\frac{2\,(\gamma-1)+\nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y+\nu\,(2-\gamma)}\right)^{\frac{\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)}{(\nu+2)[2(\gamma-1)+\nu](\gamma-2)}}, & \gamma \neq 2\,; \\ \left(\frac{2}{y+1}\right)^{\frac{2\nu}{\nu+2}}\exp\left(\frac{2\nu}{\nu+2}\,\frac{1-y}{y}\right), & \gamma = 2\,. \end{cases} \tag{33}$$

Knowing $Z(y)$ and $\xi(y)$, from the first equation of (21), we can determine the profile of the specific volume

$$V = \begin{cases} y^{-\frac{2}{\gamma-2}}\left(\frac{\gamma y+1}{\gamma+1}\right)^{\frac{2(\nu-1)}{(\gamma-1)\nu+2}}\left(\frac{2\,(\gamma-1)+\nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y+\nu\,(2-\gamma)}\right)^{-\frac{\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)}{(\gamma-2)[2(\gamma-1)+\nu][(\gamma-1)\nu+2]}}, & \gamma \neq 2\,; \\ y^{\frac{2-\nu}{\nu+2}}\left(\frac{2y+1}{3}\right)^{\frac{2(\nu-1)}{\nu+2}}\exp\left(-\frac{2\nu}{\nu+2}\,\frac{1-y}{y}\right), & \gamma = 2\,. \end{cases} \tag{34}$$

The distribution of the dimensionless pressure is found from expression (20):

$$P = \begin{cases} y^{\frac{\gamma}{\gamma-2}}\left(\frac{\gamma+1}{2}\right)^{\frac{2\nu}{\nu+2}}\left(\frac{\gamma+1}{\gamma y+1}\right)^{\frac{2\gamma(\nu-1)}{(\gamma-1)\nu+2}}\left(\frac{2\,(\gamma-1)+\nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y+\nu\,(2-\gamma)}\right)^{\frac{\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)}{(\nu+2)(\gamma-2)[(\gamma-1)\nu+2]}}, & \gamma \neq 2\,; \\ y^{\frac{2(\nu-2)}{\nu+2}}\left(\frac{y+1}{2}\right)^{\frac{2\nu}{\nu+2}}\left(\frac{3}{2y+1}\right)^{\frac{4(\nu-1)}{\nu+2}}\exp\left(\frac{2\nu}{\nu+2}\,\frac{1-y}{y}\right), & \gamma = 2\,. \end{cases} \tag{35}$$

To determine the coordinate η we must use the third relation from (21) in the case $\nu \neq 1$ and formula (22) in the case of a plane explosion. Finally we obtain the following dependence (true for any ν) for the dimensionless Eulerian coordinate:

$$\eta = \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\nu+2}}\left(\frac{\gamma y+1}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)\nu+2}}\left(\frac{2\,(\gamma-1)+\nu\,(3-\gamma)}{[2\,(\gamma-1)+\nu]\,y+\nu\,(2-\gamma)}\right)^{\frac{\gamma^2(\nu^2+4)-\gamma(3\nu^2-8\nu+4)+4\nu(\nu-2)}{(\nu+2)[2(\gamma-1)+\nu][(\gamma-1)\nu+2]}}\,. \tag{36}$$

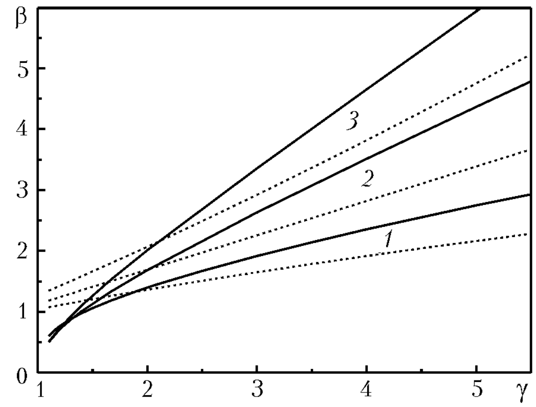

Fig. 1. Dimensionless parameter β vs. symmetry factor ν and adiabatic exponent γ: 1, 2, and 3) for ν = 1, 2, and 3 respectively. Solid curves, calculation according to (37), dashed curves, rough estimate (38).

Relations (30), (31), and (33)–(36) completely determine the parametric solution of self-similar strong-explosion equations. The constant β appearing in (12) is found from the law of conservation of energy (13):

$$\beta = \left\{ \frac{\sigma_\nu}{\nu} \frac{2}{(\gamma+1)^2} \left(\frac{2}{\nu+2} \right)^2 \int_0^1 (PV + U^2)\, d\xi \right\}^{-\frac{\nu}{\nu+2}}. \quad (37)$$

Discussion of the Results. The results of calculation of the dimensionless parameter β as a function of γ for different ν values are presented in Fig. 1. This quantity can roughly be evaluated, if it is assumed that the entire mass of the gas involved in motion has the parameters corresponding to the shock-wave front. Then we have $P = V = U = 1$ and

$$\beta = \left\{ \frac{16\sigma_\nu}{(\gamma+1)^2 \nu (\nu+2)^2} \right\}^{-\frac{\nu}{\nu+2}}. \quad (38)$$

Analyzing expressions (33)–(35), we should note that, despite the apparent difference in the behavior of the solution for $\gamma \neq 2$ and $\gamma = 2$, its dependence on the quantity γ is continuous. This is yielded by the above formulas (33)–(35): if we carry out the expansion $\gamma = 2 + \delta$ in the first of them and pass to the limit $\delta \to 0$, we obtain the second corresponding expressions.

The velocity U and the Eulerian coordinate η must vanish when $\xi = 0$. According to (30), (33), and (36), this condition corresponds to the values of the parameter $y \to \infty$. Thus, the gasdynamic profiles in the region of motion are described with variation in y in the interval [1, ∞]. The asymptotic behavior of the solution near the epicenter of the explosion (small ξ) has the form

$$U \sim \xi^{\frac{\gamma-1}{\nu\gamma}}, \quad Z \sim \xi^{-\frac{1}{\gamma}}, \quad V \sim \xi^{-\frac{1}{\gamma}}, \quad P \sim \xi^0, \quad \eta \sim \xi^{\frac{\gamma-1}{\nu\gamma}}. \quad (39)$$

The characteristics of flow in strong explosion as functions of the self-similar variable ξ for a spherical explosion are shown in Fig. 2.

We note that a singularity in the solution occurs for $\nu = 3$ and the adiabatic exponent $\gamma \geq 7$. Indeed, if $\gamma \to 7$, the expressions in the last brackets in (30), (31), and (33)–(36) are equal to $(7-\gamma)/[15(\gamma-1)]$, whence it follows that y takes on the unique value $y = 1$. The distribution of the parameters of flow along the Lagrangian mass coordinate in this case is described by the simplest relations

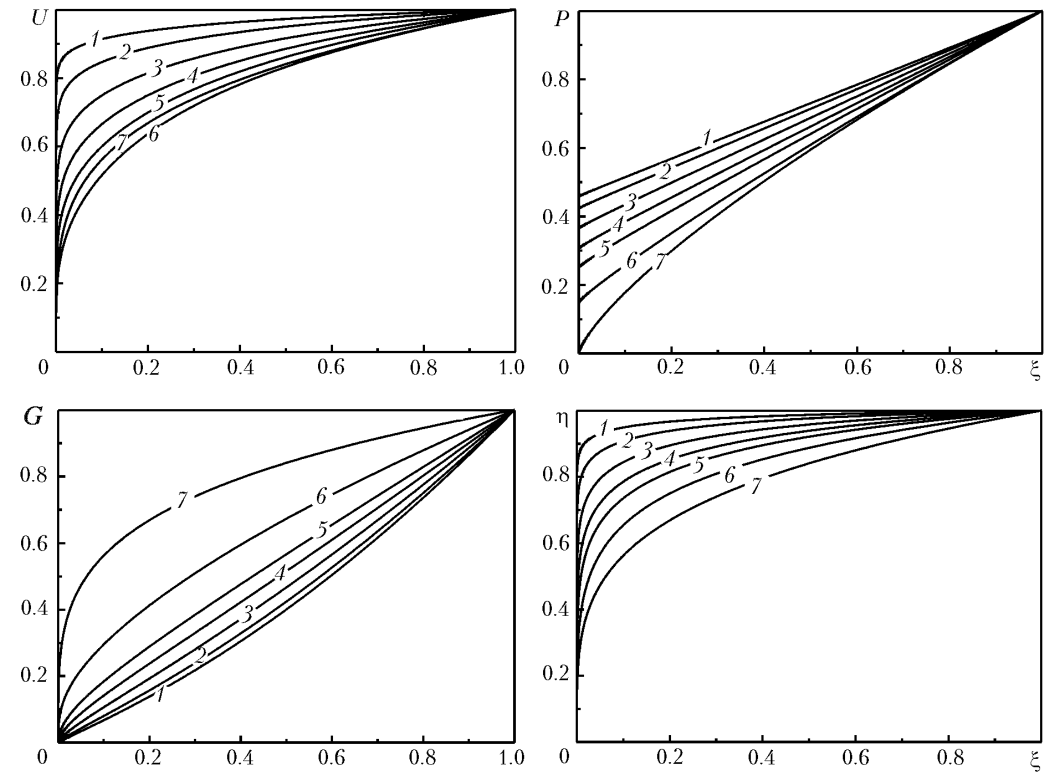

Fig. 2. Self-similar profiles of mass velocity, pressure, density, and radial component for a spherical explosion for different values of the adiabatic exponent: 1) γ = 1.1; 2) 1.2; 3) 7/5; 4) 5/3; 5) 2; 6) 3; 7) 7.

$$U=\xi^{1/4}, \quad Z=\xi^{1/2}, \quad V=\xi^{-1/4}, \quad P=\xi^{3/4}, \quad \eta=\xi^{1/4}, \tag{40}$$

which yield the Eulerian profile $U = \eta$, $Z = \eta^2$, $V = \eta^{-1}$, $P = \eta^3$, and $\xi = \eta^4$. If $\gamma > 7$, the parameter y varies in the interval $0 \le y \le 1$ and the gasdynamic profiles in this case have the following asymptotics for $\xi \to 0$:

$$U=U_0+U_1\xi^{(\gamma-2)/\gamma}, \quad Z=\xi^{(\gamma-2)/\gamma}, \quad V=\xi^{-2/\gamma}, \quad P=\xi, \quad \eta=\eta_0+\eta_1\xi^{(\gamma-2)/\gamma}. \tag{41}$$

Here U_0, U_1, η_0, and η_1 are constants easily found from (30) and (36). From expression (41) it is seen that a vacuum cavity appears at the center of a spherical explosion in the case $\gamma > 7$.

Conclusions. Strong-explosion theory has a very wide range of application [7, 8, 12], describing explosion phenomena of a different physical nature and different scale. The self-similar solution of the problem of a strong explosion, being the intermediate asymptotics, describes the stage of actual explosion processes where the mass of the moving gas is much larger than the mass of explosion products, and the pressure on the shock-wave front is much higher than the ambient pressure.

NOTATION

D, velocity of the shock-wave front, m/sec; *E*, explosion energy, J/m$^{(3-\nu)}$; *G*, *P*, *U*, *V*, and *Z*, dimensionless representative functions of density, pressure, velocity, specific volume, and temperature; *m*, Lagrangian mass coordinate, kg/m$^{(3-\nu)}$; *p*, pressure, Pa; *r*, Eulerian spatial coordinate, m; *t*, time, sec; *u*, mass velocity, m/sec; *v*, specific volume, m^3/kg; *y*, dimensionless variable determining the solution; α and β, dimensionless coefficients determining the law of propagation of the shock-wave front in spatial and mass coordinates respectively; γ, adiabatic exponent of the gas; ε, specific internal energy, J/kg; η, self-similar spatial coordinate; ν, parameter of symmetry of the problem; ξ, self-similar Lagrangian variable; ρ, density, kg/m^3; σ_V, symmetry factor relating the spatial and mass coordinates. Subscripts: f, parameters of the gas on the shock-wave front; 0, initial parameters of the gas.

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